**Tests about a Population Mean**

**Example:** Carol’s Cereal Company has been accused of under filling (on average) its 20 oz boxes of toasted oats. The evidence: A sample of 100 boxes produced an average weight of 19.9 oz with a standard deviation of 0.5 oz. How strong is the evidence? It is possible, by chance, that a random sample of 100 boxes averages under 20 oz when the average of all boxes is at least 20 oz? What’s the probability of getting a sample like this if the average is 20 ozs?

The parameter:

**= average weight of all 20 oz boxes of toasted oats produced by Carol’s Cereal Company.**

Null Hypothesis:

**H0 :**  **= 20**

Alternative Hypothesis:

**Ha : < 20**

If H0 is true then the quantity T99 = = has (approximately) a t-distribution with 99 degrees of freedom. For the given data, t99 = = -2. What is the p-value? It’s the probability getting a t99 of value -2 or less; i.e., P(T99 ≤ -2).

> pt(-2,99)

[1] 0.02411985

So, there is only a 2.4% chance of getting data like our sample data by chance if, on average, the boxes were being filled properly ( = 20). The evidence that H0 is false and Ha is true is very strong. So, we would conclude that Carol’s Cereal Company was under-filling on average. Could our judgment be incorrect? Yes, but ….

**General Problem:** Let  be a population mean.

H0 : 



**The Testing Procedure**

**Data:** From a random sample of size n,

: sample mean

s :sample standard deviation

test statistic :

If the sample size n is “large”, the value of the test statistic over all samples of size n has a t-distribution with df = n-1.

**The one sample t-test**

For an upper-tail alternative 

p-value = P(Tn-1 > t), where t is the observed value of the test statistic

For a lower-tail alternative 

p-value = P(Tn-1 < t), where t is the observed value of the test statistic

For a two-tail alternative 

p-value = 2P(Tn-1 >| t |), where t is the observed value of the test statistic

**Example1** Company A advertises that its gizmo parts have an average diameter of 3.00 cms. Company B purchases these parts from company A and depends on the parts meeting the advertised average diameter. If the average diameter is bigger or smaller than 3.00 cms, it would create a problem. So, Company B tests a random sample of 100 parts. It is willing to accept Company A’s claim unless the sample provides strong evidence that the claim is false. The sample mean for the 100 parts is  = 3.04 cm with a standard deviation s = .15 cm.

**The Parameter**

**= average diameter (cm) of all gizmo parts produced by Company A**

**The hypotheses** H0 :

Ha :  **≠ 3.00**

**Value of the test statistic**: **t99 = = 2.667**

**P-value**: 2P(T99 >2.667)

> 2\*(1-pt(2.667,99))

[1] 0.008940972

So the p-value is < 1%

Conclusion: **Given the small p=value, we reject the null hypothesis in favor of the alternative hypothesis. We conclude that Company A’s gizmo parts do not meet its claimed condition. So, Company B rejects Company A’s claim and cancels the order.**

**Example2** The data-frame **mtcars** contains data about 32 models of cars taken from Motor Trend 1974. The column **mpg** contains fuel efficiency values and the column **wt** contains the weights (in 1000 pounds). Assume this is a random sample of all models in 1974. The first car I owned averaged 17 miles per gallon. Does this sample provide strong evidence that the average efficiency of all models in 1974 was an improvement over my first car?

Parameter  = average model efficiency (mpg) in 1974

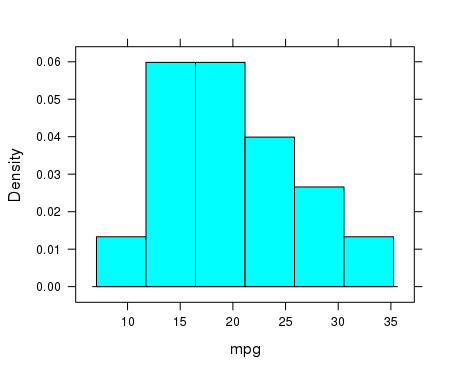
Hypotheses H0: ** = 17**

Ha : ** > 17**

Test statistic t = **, with 31 df, where**

**Preliminary Question**: Does a t-distribution fit?

* histogram(~mpg,data=mtcars)



**The distribtion is unimodal and skewed a bit to the right. The sample size n = 32 is large enough for t-based proceedures to give good results.**

**Calculating the t-statistic**

* favstats(~mpg,data=mtcars)

min Q1 median Q3 max mean sd n missing

10.4 15.425 19.2 22.8 33.9 20.09062 6.026948 32 0

**t31 = = = 2.90**

**p-value** P(T31 > 2.90)

**From R**

> > 1-pt(2.90,31)

[1] 0.003401294

**Conclusion. Since the p-value is so low, the data provides convincing evidence that the average fuel efficiency for all models in 1974 was better than the fuel efficiency for my fist car (17 mpg).**

**Letting R do ALL the work**

**General format**

t.test(dataframe$column, alternative= “greater”, “less”, “two.sided”, mu = )

> t.test(mtcars$mpg,alternative="greater", mu = 17)

One Sample t-test

data: mtcars$mpg

t = 2.9008, df = 31, **p-value = 0.003394**

alternative hypothesis: true mean is greater than 17

95 percent confidence interval:

18.28418 Inf

sample estimates:

mean of x

20.09062

**Exercises 13**

1. Test for the alternative hypothesis  < 3.4 using a sample of size 25 whose sample mean is = 3.3 and sample standard deviation is s = 0.7.
2. State the hypotheses.
3. What is the value of the test statistic?
4. Use R and pt to find the p-value.
5. Based on the data, would you reject the null hypothesis in favor of the alternative or keep the null hypothesis?
6. Was the average weight of all 1974 automobile models greater than 2900 pounds? Use the data in mtcars to test this. Does the data in mtcars provide strong evidence that the average weight for all 1974 models is greater than 2900 pounds?
7. Define the parameter.
8. State the hypotheses. **Note that the weight units used in mtcars is 1000 pounds, not pounds.**
9. Check whether using a t-distribution is acceptable by plotting a histogram of the data. Include the histogram as part of your answer to (d).
10. What is the value of the test statistic t? Use R and the function pt to find the p-value. (1-pt(t,31) will do the job)
11. Find the p-value of the data, letting R do all of the work. Include the R-command you used and the R-output.
12. Based on the p-value, what is your conclusion about the average of all model weights in 1974?
13. The World Health Organization (WHO) has issued a preliminary guideline for lead concentration in drinking water. It states that the average concentration should be at less than 10 g/l (micrograms/liter). Company A produces drinking water. It must test its water to determine whether it meets the WHO preliminary guideline. Let μ = average concentration of lead in its drinking water. There are two possible ways of setting up the hypotheses.

Hypotheses 1 Hypotheses 2

H0 : μ = 10 H0 : μ = 10

Ha : μ < 10 Ha : μ >10

For Hypotheses 1, the default assumption is that the company’s water does not meet the guideline and the company’s water will not pass unless the data provides strong evidence that it does meet the guideline.

For Hypotheses 2, the default assumption is that the company’s water does meet the guideline and it will pass unless the data provides strong evidence that it does not meet the guideline.

If you were the company, which of these two sets of hypotheses would you prefer?

If you were a consumer, which would you prefer?

1. The data-frame **warpbreaks** contains data on the number of warpbreaks per loom, where a loom corresponds to a fixed length of yarn. The column **breaks** contains the number of breaks. There are 54 data points. Does this data provide strong evidence that the average number of breaks per loom for all looms is less than 31?
2. State the hypotheses, where μ is the average number of breaks per loom for all looms.
3. Produce a histogram of the data to check whether a t-test is appropriate.
4. Use R to find the mean and standard deviation of the data and use those values to compute (by hand and calculator) the value of the test statistic t53. Use R and the function pt to find the p-value.
5. Let R do all of the work in finding the p-value. Show your R command and the output.
6. What is your conclusion about the hypotheses?